# Improving quantum cryptography with computational assumptions

Alex Bredariol Grilo





#### Quantum helps malicious parties



Quantum helps honest parties

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How do quantum resources allow us to achieve better cryptographic protocols?

Quantum mechanics					
Q	uantum states	Evolution	Measurements		

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	Basis
$ \phi_1\rangle =  +\rangle$	7
$ \phi_2 angle= 0 angle$	$\rightarrow$
$\ket{\phi_3}=\ket{1}$	$\rightarrow$
$ \phi_4 angle =  0 angle$	$\rightarrow$
$ \phi_5 angle =  - angle$	$\nearrow$
$ \phi_6 angle =  - angle$	$\nearrow$















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## Beyond QKD



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#### What if we use computational assumptions?

- Quantum protocol for multi-party computation from weaker computational assumptions
- Improving the round complexity of QKD

Quantum protocol for multi-party computation from weaker computational assumptions



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Theorem [MMP'12]

MPC cannot be built from OWF in a black-box way

#### Ideal functionality









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#### Quantum protocol for MPC from OWF

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# Corollary Quantum protocol for MPC from OWF vs. Classical protocols require PKE assumptions

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- DGILYY'24+IYYLGD'25: Implementing quantum OT in the lab

#### Corollary

```
(Practical?) Quantum protocol for MPC from OWF
vs.
Classical protocols require PKE assumptions
```











$$\vec{x} \in \{0,1\}^{\lambda}$$

$$\vec{\theta} \in \{\rightarrow,\nearrow\}^{\lambda}$$

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$$\vec{\theta} \in \{\rightarrow,\clubsuit\}^{\lambda}$$

$$\downarrow \text{Measurement}$$

$$\vec{x} \in \{0,1\}^{\lambda}$$

$$I_{b} = \{i:\theta_{i} = \hat{\theta}_{i}\}$$

$$I_{\overline{b}} = \{i:\theta_{i} \neq \hat{\theta}_{i}\}$$

$$\vec{x} \in \{0,1\}^{\lambda}$$

$$\vec{\theta} \in \{\rightarrow,\nearrow\}^{\lambda}$$

$$a_{0} = Enc_{\vec{x}_{l_{0}}}(m_{0})$$

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Attack for malicious receiver:  $\tilde{R}$  waits  $\vec{\theta}$  to measure the qubits using the right basis

#### Bit-commitment with simulation security


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 $|x_{\theta^1}^1\rangle|x_{\theta^2}^2\rangle...|x_{\theta^\lambda}^\lambda\rangle$  $\vec{\hat{\theta}} \in \{\rightarrow, \nearrow\}^{\lambda}$  $ec{x} \in \{0,1\}^{\lambda}$  $ec{ heta} \in \{
ightarrow, 
earrow \}^{\lambda}$ ↓ Measurement  $ec{x} \in \{0,1\}^\lambda$ S R  $\vec{\theta}$  $I_b = \{i : \theta_i = \hat{\theta}_i\}$  $I_0, I_1$  $I_{\overline{b}} = \{i : \theta_i \neq \hat{\theta}_i\}$  $a_0 = Enc_{\vec{x}_{l_0}}(m_0)$  $a_1 = Enc_{\vec{x}_{l_1}}(m_1)$  $a_0, a_1$  $m_b = Dec_{\vec{x}_{l_i}}(a_b)$ 

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_		[BCKM21]		[GLSV21]
_	<ol> <li>(Black-box) equivocality compiler</li> <li>Extractable commitment from equivocal commitment and quantum communication</li> </ol>	1.	Equivocal commitment from Naor's commitment and zero-knowledge	
		equivocal commitment and quantum communication	2.	Unbounded-simulator OT from equivocal commitment
			3.	Extractable and equivocal commitment from unbounded-simulator OT and quantum communication
Features:				
	•	Black-Box use of one-way functions	•	Constant-Round OT in the CRS model
	•	Statistical security against malicious receiver	•	Statistically binding extractable commitment





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#### We have a protocol that only uses BB84 states. Great, let's implement it!









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Hmmm.. OK. How many states do you need to send?





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The security parameter.

What is  $\lambda$ ?





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 $\begin{array}{c} {\sf Hmmm.} \ {\sf OK}.\\ {\sf How many states do you need to send?}\\ {\sf poly}(\lambda) \end{array}$ 

What is  $\lambda$ ?

The security parameter. How many bits of security do you achieve?



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How many bits of security do you achieve?  $\operatorname{\mathsf{poly}}(\lambda)$ 



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How many bits of security do you achieve?  $poly(\lambda)$ 

This  $\lambda$  again... And the classical post-processing?



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 $poly(\lambda)$ This  $\lambda$  again... And the classical post-processing? No idea how to implement it.



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- Difficulties in implementation
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# Practical OT from OWF

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Practical protocol [DGILYY'24] ↔ Experimental implementation [IYYLGD'24 – on-going]

	BCKM21 numerical analysis	DGILYY24 implementation profiling
N <sub>BB84</sub>	$1.7\cdot 10^{13}$	$2.1 \cdot 10^6$
N <sub>RNG</sub>	$1.3\cdot 10^{16}$	$1.5 \cdot 10^9$
N <sub>PRG</sub>	$6.6\cdot10^{15}$	$1.7 \cdot 10^{7}$
M <sub>seed</sub>	8.5 · 10 <sup>17</sup> bytes	2.5 · 10 <sup>10</sup> bytes
$T_{acqBB84}$	197 days	2.1 s
T <sub>exec</sub>	_	13 min 5 s

### Implementation





## Improving the round complexity of QKD

### QKD

- quantum communication: technologically challenging but round-efficient
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  - sifting
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QKD with simultaneous messages and simple quantum states

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  - extra round of simultaneous messages to have indistinguishability security

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- **③** Classically shared key is used for choosing the basis measurement of the EPR pair

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- Security: reduction to attack to classical KA
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  - Offline attacker characterizes the structure of the entanglement shared between Alice, Bob and Eve
  - Alice and Bob's quantum state in the offline phase is sufficient for breaking classical KA

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# Thank you for your attention!